USN | | | | | | | | | | **10MAT31**

Third Semester B.E. Degree Examination, June/July 2018
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Obtain the Fourier Series for the function,

$$f(x)=\begin{cases}\pi x & \text{in } 0\le x\le 1\\ \pi(2-x) & \text{in } 1\le x\le 2\end{cases}.$$ **(07 Marks)**

b. Find the cosine half range series for $f(x)=x(l-x)$; $0\le x\le l$. **(06 Marks)**

c. Obtain the Fourier series of y upto the second harmonics for the following values: **(07 Marks)**

x^0	45	90	135	180	225	270	315	360
y	4.0	3.8	2.4	2.0	-1.5	0	2.8	3.4

2 a. Find the Fourier transform of $f(x)=e^{-|x|}$. **(07 Marks)**

b. Find the Fourier sine transform of $f(x)=\dfrac{1}{x(1+x^2)}$. **(06 Marks)**

c. Find the Fourier cosine transform of e^{-ax} and deduce that

$$\int_0^\infty \frac{\cos mx}{a^2+x^2}dx=\frac{\pi}{2a}e^{-am}.$$ **(07 Marks)**

3 a. Obtain the various possible solution of one-dimensional wave equation $u_{tt}=C^2u_{xx}$ by the method of separation of variables. **(07 Marks)**

b. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If each of its points is given a velocity $\lambda x(l-x)$. Find the displacement of the string at any distance x from one end at any time t. **(06 Marks)**

c. Solve the Laplace equation, $\dfrac{\partial^2u}{\partial x^2}+\dfrac{\partial^2u}{\partial y^2}=0$

subject to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\dfrac{n\pi x}{l}$. **(07 Marks)**

4 a. Predict the mean radiation dose at an altitude of 3000 feet by fitting an exponential curve to the given data using $y=ab^x$ **(07 Marks)**

Altitude (x) :	50	450	780	1200	4400	4800	5300
Dose of radiation (y) :	28	30	32	36	51	58	69

b. Using graphical method solve the LPP,

Maximize $z=50x_1+60x_2$,

Subject to the constraints : $2x_1+3x_2\le 1500$,

$3x_1+2x_2\le 1500$,

$0\le x_1\le 400$,

$0\le x_2\le 400$,

$x_1\ge 0,\ x_2\ge 0$. **(06 Marks)**

c. Solve the following minimization problem by simplex method:
Objective function : $P = -3x + 8y - 5z$
Constraints : $-x - 2z \le 5$,
$2x - 3y + z \le 3$,
$2x - 5y + 6z \le 5$,
$x_1, x_2, x_3 \ge 0$. **(07 Marks)**

PART – B

5 a. Using Newton-Raphson iterative formula find the real root of the equation $x \log_{10} x = 1.2$. Correct to five decimal places. **(07 Marks)**

b. Solve, by the relaxation method, the following system of equations:
$9x - 2y + z = 50$
$x + 5y - 3z = 18$
$-2x + 2y + 7z = 19$. **(06 Marks)**

c. Using the Rayleigh's power method find the dominant eigen value and the corresponding eigen vector of the matrix, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ taking $[1,\ 1,\ 1]^T$ as the initial eigen vector. Peform five iterations. **(07 Marks)**

6 a. The population of a town is given by the table. Using Newton's forward and backward interpolation formulae, calculate the increase in the population from the year 1955 to 1985. **(07 Marks)**

Year	1951	1961	1971	1981	1991
Population in thousands	19.96	39.65	58.81	77.21	94.61

b. The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9, 10 of the independent variable. What is the best estimate you can give for the value of the function at the position 6 of the independent variable? Use Lagrange's method. **(06 Marks)**

c. Use Simpson's $\left(\frac{3}{8}\right)^{th}$ Rule to obtain the approximate value of $\int_0^{0.3} \left(1 - 8x^3\right)^{\frac{1}{2}} dx$ by considering 3 equal intervals. **(07 Marks)**

7 a. Solve numerically the wave equation $u_{xx} = 0.0625u_{tt}$ subject to the conditions, $u(0, t) = 0 = u(5, t)$, $u(x, 0) = x^2(x - 5)$ and $u_t(x, 0) = 0$ by taking $h = 1$ for $0 \le t \le 1$. **(07 Marks)**

b. Solve : $u_{xx} = 32u_t$ subject to the conditions, $u(0, t) = 0$, $u(1, t) = t$ and $u(x, 0) = 0$. Find the values of u upto t = 5 by Schmidt's process taking $h = \frac{1}{4}$. Also extract the following values:
(i) u(0.75, 4) (ii) u(0.5, 5) (iii) u(0.25, 4) **(06 Marks)**

c. Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square region shown in the following Fig. Q7 (c), with the boundary values as indicated in the figure. Carry out two iterations. **(07 Marks)**

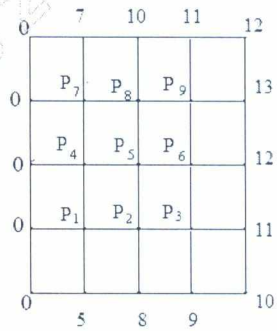

Fig. Q7 (c)

8 a. State initial value property and final value property. If $\bar{u}(z) = \frac{2z^2 + 3z + 4}{(z-3)^3}$, $|z| > 3$. Find the values of u_1, u_2, u_3. **(07 Marks)**

b. Obtain the inverse z-transform of the function, $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$. **(06 Marks)**

c. Solve the difference equation, $y_{n+1} + \frac{1}{4} y_n = \left(\frac{1}{4}\right)^n$, $(n \geq 0)$, $y_0 = 0$ by using z-transform method. **(07 Marks)**

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10ME/AU33

Third Semester B.E. Degree Examination, June/July 2018
Basic Thermodynamics

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO full questions from each part.***
2. Use of Thermodynamic data hand book is permitted.

PART – A

1 a. Differentiate between the following with suitable examples:
i) Intensive and extensive properties.
ii) Path and point functions. **(06 Marks)**

b. Explain the concept of temperature measurement using thermocouple. **(04 Marks)**

c. State the Zeroth law of thermodynamics and briefly explain its significance. **(04 Marks)**

d. A temperature scale is being developed using the following relation $t = a\,ln(P) + \left(\frac{b}{2}\right)$ where 'P' is the thermometric property and 'a' and 'b' are constants. Determine Celsius temperature corresponding to thermometric property of 6.5, if ice point and steam point give thermometric property value of 3 and 8. **(06 Marks)**

2 a. Define heat and thermodynamic definition of work. Calculate the work done by the system when the gas expands reversibly to a volume V_2 m^3 during the following processes:
i) Isothermal process ii) Polytropic process. The initial pressure and volume of a mass of gas in a cylinder fitted with a movable piston are P_1 bar and V_1 m^3 respectively. **(12 Marks)**

b. A cylinder contains one kg of fluid at an initial pressure of 20bar. The fluid is allowed to expand reversible behind a piston according to law $PV^2 = C$ until the volume is doubled. The fluid is then cooled reversibly at constant pressure until the piston regains its original position, heat is then supplied reversibly with the piston firmly locked in this position until the pressure rises to the original value of 20bar. Calculate the net work done by the fluid for an initial volume of $0.05m^3$. **(08 Marks)**

3 a. State first law of thermodynamics for a closed system undergoing a cycle process. Show that internal energy is the property of the system. **(06 Marks)**

b. Derive steady flow energy equation for a single stream of fluid entering and learning the control volume. **(06 Marks)**

c. Air enters a gas turbine with velocity 105 m/s, specific volume $0.8m^3/kg$ and leaves at 135m/s and $1.5m^3/kg$. The inlet area of the gas turbine is $0.05m^2$. As air passes through the turbine, the specific enthalpy decreases by 145 kJ/kg and air loses 27 kJ/kg of heat. Determine: i) Mass flow rate of air ii) Exit area of the turbine iii) Power developed by the turbine. **(08 Marks)**

4 a. Write two statements of second law of thermodynamics and show their equivalence. **(08 Marks)**

b. Explain the various causes of irreversibility. **(04 Marks)**

c. A reversible heat engine extracts heat from three reservoirs at 1000K, 810K and 595K. The engine delivers 10×10^3 J/S of network and rejects 400 kJ/min of heat to a sink at 298K. If the heat supplied to the reservoir at 1000K is 55% of the heat supplied by the reservoir at 595K. Determine quantity of heat absorbed by each reservoir. **(08 Marks)**

PART – B

5 a. State and prove Clausius inequality. **(06 Marks)**

b. Show that entropy is a property. **(04 Marks)**

c. Define available and unavailable energy. **(04 Marks)**

d. A reversible engine extracts 75kW of energy from a reservoir at 750K and produces 15kW of work. The engine rejects heat to two reservoirs at 650K and 58 K respectively. Determine quantity of heat rejected to each sink. **(06 Marks)**

6 a. Draw a P-T diagram for pure substance and indicate all necessary points and different regions on it. **(06 Marks)**

b. Define dryness fraction. With a neat sketch, explain the measurement of dryness fraction of steam by using separating and throttling calorimeter. **(08 Marks)**

c. A rigid vessel contains liquid-vapour mixture in the ratio of 3:2 by volume. Determine quality of water vapour mixture and total mass of fluid in vessel, if the volume of vessel is $2m^3$ and initial temperature is 150°C. **(06 Marks)**

7 a. Write down the Td_s equations and derive the expression for the difference in heat capacities C_p and C_v. **(10 Marks)**

b. $1.2m^3$ of air is heated reversibly at constant pressure from 300K to 600K and is then cooled reversibly at constant volume back to initial temperature. If the initial pressure is 1 bar calculate:

i) The net heat flow

ii) The overall change in entropy.

Represent the processor T-S plot. Take C_p = 1.005 kJ/kg K and R = 0.287 kJ/kg K. **(10 Marks)**

8 a. Show that the entropy change of an ideal gas is given by the equation of the form $S_2 - S_1 = C_p \, ln\frac{V_2}{V_1} + C_v \, ln\frac{P_2}{P_1}$. **(08 Marks)**

b. State and explain Dalton's law of partial pressures and Amagat's law of additive volumes. **(08 Marks)**

c. One kg of CO_2 has a volume of $1m^3$ at 100°C. Compute the pressure by i) Vander Walls equation ii) Perfect gas equation. **(04 Marks)**

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10ME34/10AU34

Third Semester B.E. Degree Examination, June/July 2018
Mechanics of Materials

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Draw stress-strain diagram, for a mild steel subjected to tension and indicate salient points on the diagram. **(06 Marks)**

b. Define : (i) Nominal stress (ii) True stress (iii) Hook's law. **(04 Marks)**

c. A member is subjected to point loads as shown in Fig. Q1 (c). Calculate P_2 necessary for equilibrium. Take $E = 2.05 \times 10^5 \text{ N/mm}^2$. **(10 Marks)**

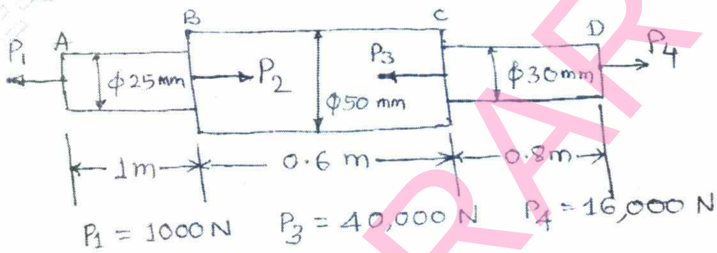

Fig. Q1 (c)

2 a. Define : (i) Poisson's ratio (ii) Modulus of rigidity (iii) Bulk modulus and (iv) Volumetric strain. **(04 Marks)**

b. Establish the relationship between modulus of elasticity and Bulk modulus in case of a cube subjected to three mutually perpendicular like compressive stresses of equal intensity 'P'. **(06 Marks)**

c. A composite bar is rigidly fitted at the supports A and B as shown in Fig. Q2 (c). Determine the reactions at supports when temperature rises by 20°C. Take E_a = 70 GN/m², E_S = 200 GN/m², $\alpha_a = 11 \times 10^{-6} /^\circ C$, $\alpha_S = 12 \times 10^{-6} /^\circ C$ **(10 Marks)**

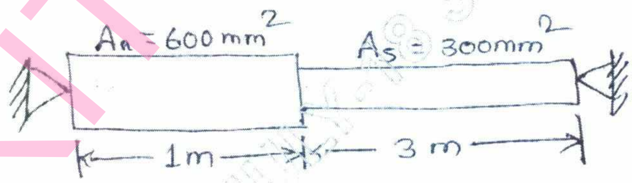

Fig. Q2 (c)

3 a. The state of stress at a point in a strained material is as shown in Fig. Q3 (a). Determine :
(i) The magnitude of principal stresses.
(ii) The direction of principal stresses and
(iii) The magnitude of the maximum shear stress and its direction.
Indicate all the planes by a sketch. **(10 Marks)**

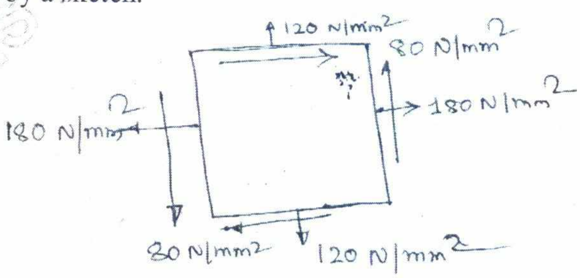

Fig. Q3 (a)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

b. The direct stresses at a point in a strained material are 100 N/mm^2 and 60 N/mm^2 as shown in Fig. Q3 (b). Determine stress on the inclined plane AC. **(10 Marks)**

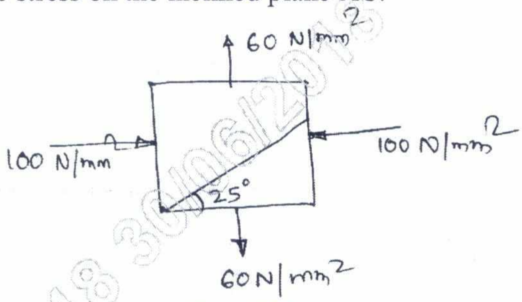

Fig. Q3 (b)

4 a. Define strain energy and Resilience. **(02 Marks)**

b. Two bars, each of length L and of different materials are each subjected to the same tensile force P. The first bar has a uniform diameter 'D' and the second bar has a diameter of $\frac{D}{2}$ for a length $\frac{L}{4}$ and a diameter D for the remaining length. Compare the strain energies of the two bars if, (i) $\frac{E_1}{E_2} = \frac{4}{7}$ and (ii) $E_1 = E_2$ **(08 Marks)**

c. A thin cylindrical shell 2 m long has 200 mm diameter and thickness of metal 10 mm. It is filled completely with a fluid at atmospheric pressure. If an additional fluid of 25000 mm^3 is pumped in, find the pressure developed. Find also the changes in diameter and length. Take $E = 2 \times 10^5$ N/mm^2 and $\mu = 0.3$. **(10 Marks)**

PART – B

5 a. With neat sketches explain: (i) Types of beams. (ii) Types of loads (iii) Types of supports. **(06 Marks)**

b. The simply supported beam shown in Fig. Q5 (b), carries two concentrated loads and a uniformly distributed load. Draw shear force diagram and bending moment diagram. **(14 Marks)**

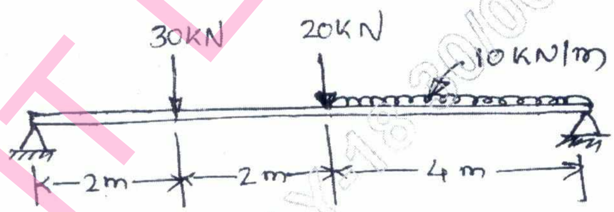

Fig. Q5 (b)

6 a. State the assumptions made in simple theory of bending. **(04 Marks)**

b. A circular pipe of external diameter 70 mm and thickness 8 mm is used as a simply supported beam over an effective span of 2.5 m. Find the maximum concentrated load that can be applied at the centre of the span if permissible stress in tube is 150 N/mm^2. **(08 Marks)**

c. A wooden section $300\text{mm} \times 300\text{mm}$ has a central bore of 100 mm diameter as shown in Fig. Q6 (c). If it is used as a beam to resist a shear force of 10 kN, find the shearing stress at crown of the bore and at neutral axis. **(08 Marks)**

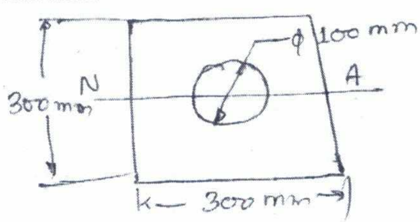

Fig. Q6 (c)

7 a. Find displacement at free end of the Cantilever beam shown in Fig. Q7 (a). Take $E = 2\times10^5 N/mm^2$ and $I = 2\times10^8 mm^4$. **(08 Marks)**

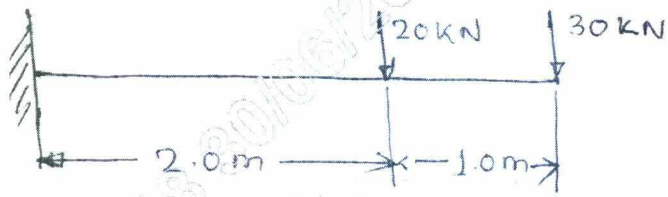

Fig. Q7 (a)

b. A simply supported beam of 6 m span is subjected to a set of loads as shown in Fig. Q7 (b). Find maximum deflection and the maximum slope for the beam.
Take $EI = 15\times10^9$ KN-mm^2. Use Macanlay's method. **(12 Marks)**

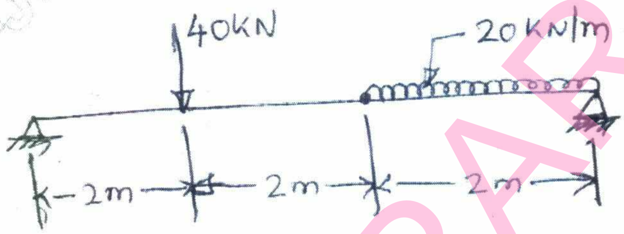

Fig. Q7 (b)

8 a. Determine the diameter of solid shaft which transmits 440 kW at 280 rpm. The angle of twist must not exceed one degree per meter length and the maximum shear stress is to be limited to 40 N/mm^2. Assume G = 84 KN/mm^2. **(10 Marks)**

b. A 2 m long pin ended column of square cross section is to be made of wood. Assuming E = 12 GPa and allowable stress being limited to 12 MPa. Determine the size of the column to support the following loads (i) 95 kN (ii) 200 kN. **(10 Marks)**

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10MEA/AUA302

Third Semester B.E. Degree Examination, June/July 2018

Material Science & Metallurgy

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Illustrate the following point defects which disrupt the perfect arrangement of the surrounding atoms in a crystal structure. **(15 Marks)**
(i) Vacancy (ii) Interstitial atom (iii) Small substitutional atom
(iv) Large substitutional atom. (v) Frenkel defect.

b. What is atomic diffusion? Mention any 3 examples of diffusion. **(05 Marks)**

2 a. Discuss how the stress-strain behavior of iron varies with temperature. **(12 Marks)**

b. An aluminum specimen originally 300 mm long is pulled in tension with a stress of 280 MPa. If the deformation is entirely elastic, what will be the resultant elongation? E for Aluminum is 69 GPa. **(03 Marks)**

c. A tensile stress is applied along the longitudinal direction of a cylindrical aluminum rod that has a diameter of 10 mm. Determine the magnitude of the load required to produce a 2.5×10^{-3} mm change in diameter, if the deformation is entirely elastic. E for aluminum is 69 GPa, Poisson's ratio for A*l* is 0.33. **(05 Marks)**

3 a. Present a schematic representation of the typical constant load creep behavior of metals and discuss. **(08 Marks)**

b. What is fatigue limit? Also, discuss the stress amplitude (s) versus logarithm of the number of cycles (N) to fatigue failure of metals for, (i) a material that displays a fatigue limit and (ii) a material that does not display a fatigue limit. **(12 Marks)**

4 a. Explain with necessary diagrams, how the macrostructure (ingot structure) of a casting develops during solidification. **(12 Marks)**

b. State the Gibbs phase rule. **(02 Marks)**

c. Explain the Hume-Rothery rules for extensive solid solubility of one element in another. **(06 Marks)**

PART – B

5 a. Illustrate the microstructures for an iron-carbon alloy of eutectoid composition, above and below the eutectoid temperature. **(12 Marks)**

b. Determine the composition of each phase in a Cu-40% Ni alloy at 1300°C, 1270°C, 1250°C and 1200°C (Use Fig. Q5 (b)). **(08 Marks)**

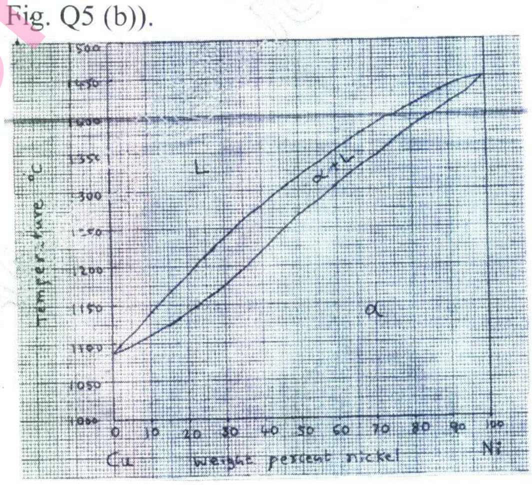

Fig. Q5 (b)

6 a. Illustrate the formation of quench cracks in steels, when they are quenched. Also, discuss the marquenching heat treatment designed to reduce residual stresses and quench cracking. **(10 Marks)**

b. Illustrate the setup for the Jominy test used for determining the hardenability of steel. Also show hardenability curves for several steels. **(10 Marks)**

7 a. Show schematically the microstructures of the following types of cast iron : Gray iron, White iron, Malleable iron, ductile iron and compacted graphite iron. **(10 Marks)**

b. List the properties and applications of copper and aluminum alloys. **(10 Marks)**

8 a. What are composite materials? How they are classified? **(07 Marks)**

b. Illustrate the following production methods:
   (i) Hand lay-up method for molding fiber reinforced plastic.
   (ii) Filament winding process for producing fiber-reinforced plastic composite material. **(10 Marks)**

c. Schematically represent the following types of fiber reinforced composites:
   (i) Continuous and aligned fibers.
   (ii) Discontinuous and aligned fibers.
   (iii) Discontinuous and randomly oriented fibers. **(03 Marks)**

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10MEB306/10AUB306

Third Semester B.E. Degree Examination, June/July 2018
Fluid Mechanics

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. 3.6 m^3 of certain liquid weighs 24.72 kN. Calculate
(i) Specific weight (ii) Mass Density (iii) Specific volume (iv) Specific gravity (v) Dynamic viscosity in centipoise if kinematic viscosity is 7.8 stokes. (vi) The rise of liquid height in a tube of diameter 10 mm, taking surface tension, 0.0625 N/m and angle of contact, 20° with respect to glass tube. **(08 Marks)**

b. Obtain an expression for intensity of pressure due to surface tension in (i) Liquid Droplet (ii) Hollow Bubble (iii) Liquid Jet. **(06 Marks)**

c. Explain the phenomenon of cavitation and its effects. **(06 Marks)**

2 a. State and prove the Hydrostatic law. An open tank contains three immiscible liquids for the depths of 1m, 1.5m and 2m as shown in Fig.Q2(a). Determine the value of pressures at the interface points A and B and pressure at point C. **(10 Marks)**

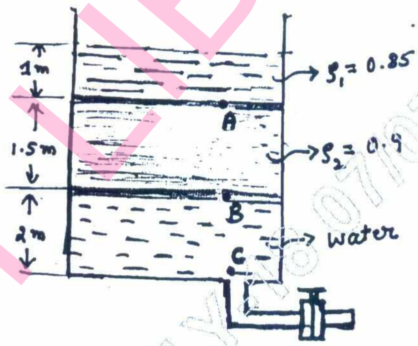

Fig.Q2(a)

b. Define Centre of pressure and Total pressure. Obtain an expression for centre pressure and Total pressure for the vertical surface immersed in a liquid. **(10 Marks)**

3 a. Define the following : (i) Buoyancy and centre of Buoyancy
(ii) Metacentre and Metacentric Height
A rectangular pontoon is 5 m long, 3 m wide and 1.2 m high. The depth of immersion of the Pontoon is 0.80 m in sea water. If the centre of gravity is 0.6 m above the bottom of the pontoon, determine the metacentric height. Take 'ρ' for sea water at 1025 kg/m^3. **(10 Marks)**

b. Define velocity potential function and stream function. The stream function for a two-dimensional flow is given by $\psi = 2xy$. Calculate the velocity at the point P(2, 3). Find the velocity potential function 'ϕ' equation. **(10 Marks)**

4 a. Derive the Bernoulli's equation of motion and state its assumptions. **(10 Marks)**

b. What do you understand by Bernoulli's equation for Real fluids? A pipe of diameter 350mm at one end is varying with a slope of 1 in 30 for a length of 150 m. The diameter at the other end is 200 mm. Calculate (i) intensity of pressure at the smaller end if the pressure at the bigger end is 40.5 N/cm^2. (ii) Determine the Head loss and direction of flow if the pressure at the smaller end is 28.5 N/cm^2. Take discharge through the pipe as 40 litres/s. **(10 Marks)**

PART – B

5 a. What is venturimeter? Obtain an expression for the rate of flow through venturimeter. **(10 Marks)**

b. What do you mean by Dimensional Homogeneity? The efficiency η of a fan depends on density ρ, dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor and the discharge Q. Express the efficiency η in terms of dimensionless parameters using Buckingham π - theorem. **(10 Marks)**

6 a. How the loss of energy in pipes is classified? **(04 Marks)**

b. Obtain the Chezy's equation for the loss of Head through pipes. **(06 Marks)**

c. Define Hydraulic Gradient-line and Total energy line. Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m. When one end of the pipe is open to the atmosphere. The pipe is horizontal and Height of water in tank is 4 m above the centre of the pipe. Take f = 0.009. **(10 Marks)**

7 a. Obtain an expression for shear stress distribution and velocity distribution for the flow of viscous fluid in a pipe. **(08 Marks)**

b. Prove that $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$ for the viscous flow between the two parallel plates. **(06 Marks)**

c. What power is required per km of a line to overcome the viscous resistance to the flow of glycerine through a horizontal pipe of 200 mm at the rate of 15 litres/s. Take μ = 8.5 poise and kinematic viscosity, ν = 6.5 stokes. **(06 Marks)**

8 a. Obtain an expression for Drag force and lift force for the flow post over the solid body. **(07 Marks)**

b. Explain the formation of a Boundary layer for the flow over a plate having free stream velocity. **(05 Marks)**

c. What is a Mach number and how is it important as a non-dimensional parameter. A projectile travels in air of pressure 10.1043 N/cm^2 at 10°C at a speed of 1500 km/hr. Find the Mach number and Mach angle. Take K = 1.4, R = 287 J/kgK. **(08 Marks)**

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2002 SCHEME

USN

MATDIP301

Third Semester B.E. Degree Examination, June/July 2018
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

1 a. Find modulus and amplitude of : $z = \frac{(1+i)^2}{1-i}$. **(06 Marks)**

b. Prove that :
$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{nH}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}$. **(07 Marks)**

c. If $x = \cos\theta + i\sin\theta$ and $y = \cos\phi + i\sin\phi$, then prove that $\frac{x-y}{x+y} = i\tan\left(\frac{\theta-\phi}{2}\right)$. **(07 Marks)**

2 a. Find the n^{th} derivative of $y = e^{ax}\cos(bx+c)$. **(06 Marks)**

b. If $y = e^{m\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. **(07 Marks)**

c. Expand $\log(1+\sin x)$ in powers of x, by using Maclaurin's theorem. **(07 Marks)**

3 a. If $z = e^{ax+by}f(ax-by)$, then show that $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$. **(06 Marks)**

b. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$. **(07 Marks)**

c. If $u = \tan^{-1}x + \tan^{-1}y$ and $v = \frac{x+y}{1-xy}$ find $\frac{\partial(u,v)}{\partial(x,y)}$ **(07 Marks)**

4 a. With usual notation, prove that $\tan\phi = r\frac{d\theta}{dr}$. **(06 Marks)**

b. Find the angle between the curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$. **(07 Marks)**

c. Find the pedal equation of the curve $r = a(1+\cos\theta)$. **(07 Marks)**

5 a. Obtain the reduction formula for $\int \sin^n x\,dx$, where n is a positive integer. **(06 Marks)**

b. Evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}}dx$. **(07 Marks)**

c. Evaluate $\int_0^{\log 2}\int_0^x\int_0^{x+y} e^{x+y+z}dz\,dy\,dx$. **(07 Marks)**

6 a. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

b. Show that $\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$. **(07 Marks)**

c. Evaluate $\int_0^{\infty} \frac{dx}{1+x^4}$ in terms of Beta functions. **(07 Marks)**

7 a. Solve $\frac{dy}{dx} = \sin(x+y)$. **(06 Marks)**

b. Solve $xdy - ydx = \sqrt{x^2+y^2}\ dx$. **(07 Marks)**

c. Solve $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$. **(07 Marks)**

8 a. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \cos 2x$. **(07 Marks)**

c. Solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^{x}\cos x$. **(07 Marks)**

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